

Time as motion, in the spirit of Aristotle.

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0. Abstract.

The kinds of time used in dealing with motions of objects in classical and relativistic mechanics can be identified with fractions of revolutions of radii of certain circles. The term 'time' could be eliminated in doing such work

1. Change, motion, and distance.

In the spirit of Aristotle, I will start with *change*. Here is a dictionary definition of 'change': "an instance of making or becoming different in some particular; a departure from a norm; a deviation from established character, sequence, or condition; a divergence from uniformity or constancy in any quality, quantity, or degree."

One kind of change that unimpaired people can perceive or imagine is *motion* of things they perceive or imagine. They can also imagine things called *places* which serve to locate such things. An example is geometric and mass points, where a *point* may be thought of, in the manner of Euclid, as "that which has no parts". People perceive and imagine things moving from places to other places. Such motions may be perceived or imagined to take place against or in a background which is not itself taken to be moving, although the backgrounds themselves may be considered to be moving, perhaps in some other stationary background. It is common to conceive of a background for motions which consists of many places between which motions of things take place. Examples are Euclidean and Minkowski spaces.

Unimpaired people can perform an activity called 'counting'. To *count* is to assign marks or concepts to things. I will call the marks 'numerals' and the concepts 'numbers'. I will use the collective term 'counters' to refer to numerals and their numbers. In particular, humans can count by assigning counters in repetitive ways, so that each thing being counted is assigned a different counter. For example, the numeral 1 or term 'one' can be assigned to a thing, and the numeral 2 or term 'two' can be assigned to a thing and another different thing. When this is done, the things will be said to have been correctly counted, and these things can be referred to as a 'pair' of things.

Counters can be combined in certain ways. For example, 1 may be added to itself and the result assigned the counter 2, 1 may be added to 2 and the result assigned the counter 3, and so on, adding 1 to each result for as long as one likes. This ability can be described by what are known as the Peano postulates. One of these postulates, that of

mathematical induction, can be taken to involve or imply an ability to continue the process of counting indefinitely.

It is possible for humans to assign a counter to a pair of places as follows. A thing called a 'standard of length' may be chosen which consists of some material lying between two different perceptible places called 'ends'. One can construct such a standard of length in such a way that the standard can be moved so that one of the perceptible ends of the standard can be *seen* but no other part of the standard can be seen, or at least any parts can barely be seen. In this case, the standard will be said to be *straight*. A straight standard is a *standard of length*. If the two ends of such a standard can be positioned in such a way that they are perceived to coincide with the two places of a pair of places on some thing, the pair of places of a standard of length are to be assigned the counter 1, and the end places of the standard and the assigned places of the other pair may be said to be '1 standard length apart' or that 'the *distance* between each pair of places is one standard length'.

Many different standards of length have been made and used, and given different names. Examples are 'meter', 'kilometer', 'centimeter', 'yard', 'mile', 'inch'. Each such standard is called a *unit* of length. Units of length are chosen so that they are related to each other using counters. For example, a centimeter and meter are made in such a way that 100 centimeters can be laid down against 1 meter in such a way that each part of the meter is covered by one and only one of the centimeters; a yard and an inch are made in such a way that 36 inches can be similarly positioned against a yard. One may speak of two places as being a number of specified units apart, or of the *distance* between two places being a number of specified units. If two places are considered to coincide, the distance between them may be assigned the term 'zero' or a numeral '0'. These may be considered as a supplement to the counters.

I have given a sketch of an experiential basis for mathematical concepts such as points, straight lines, lengths assigned to bounded segments of straight lines, and physical abstractions such as units of length. It is my view that these abstractions enable people to ignore some of the complications of dealing with what they experience by ignoring certain kinds of our experience in such a way that they can, with manipulations of these concepts and using kinds of logical reasoning, approximate with some satisfying degree of accuracy some of what they experience.

2. Time without motion and motion without time.

Aristotle says in the *Physics*: "Not only do we measure the movement by the time, but also the time by the movement, because they define each other. The time marks the movement, since it is its number, and the movement the time. We describe the time as much or little, measuring it by the movement, just as we know the number by what is numbered, e.g. the number of the horses by one horse as the unit. For we know how many horses there are by the use of the number; and again by using the one horse as unit we know the number of the horses itself. So it is with the time and the movement; for we

measure the movement by the time and vice versa. It is natural that this should happen; for the movement goes with the distance and the time with the movement, because they are quanta and continuous and divisible. The movement has these attributes because the distance is of this nature, and the time has them because of the movement. And we measure both the distance by the movement and the movement by the distance; for we say that the road is long, if the [movement of the] journey is long, and that this [the movement of the journey] is long, if the road is long – the time [is long], too, if the movement [of the journey is long], and the movement [of the journey is long], if the time [is long].” (translated by R. P. Hardie and R. K. Gaye).

In talking about time measured (or “numbered”) by motion, I will assume here that changes of position of visible objects, i.e. distances between them, can be measured without paying any attention to questions about how long it may take to perform acts of measuring. I will take motion to be measured by changes of distances of a long hand of one common kind of clock. *Since motion is in this case measured without taking into account how much some sort of “time” elapse. Motion is measured atemporally.*

3. Clocks.



There are clocks which have circular faces near the circumference of which numerals from 1 to 12 are inscribed, and which have two hands attached to the center of the circular face which are rotated by some mechanism. The mechanisms are intended to move the hands



with a regularity which approximates that of the sun’s motion relative to other stars and to positions on the surface of the earth. Sundials may be taken to be ancestors of such clocks.

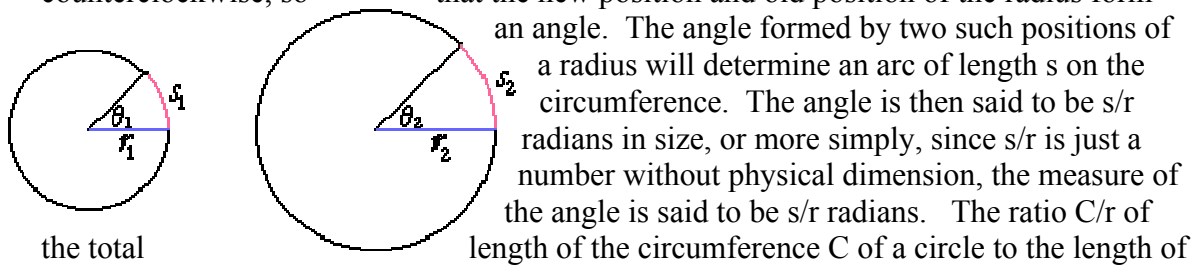
Postulate 4. The motions of celestial objects with respect to terrestrial observers can also be measured atemporally.

This is done by astronomers when they define right ascension and declination by setting an hour to be 15 degrees of arc along certain circles on an imaginary celestial sphere. Degrees of arc traversed along the circumference of a particular circle can be defined “atemporally”, as follows.

A *sector* of a circular disk is the part of the disk bounded by two radii and an included arc, as shown in the figures below. The two endpoints of the included arc may be taken to mark positions of objects, whose distances apart measured along the included arc are different for different sized circles. For example, one of the disks may be taken to represent the face of a clock, and the other a disk determined by an observer at the center of the disk who measures the angle between two positions of a celestial object along an imaginary celestial sphere centered at the observer’s position. This is a crude approximation to actual motion of, say, the sun, but it is consonant with the astronomical

techniques known to Aristotle. For example, it doesn't take into account the elliptical motion of the earth's orbit (itself subject to perturbations), and it ignores the fact that the observer is not at the center of the earth but on the earth's rotating surface. However, my point here is not to describe how "time" is measured nowadays, but to give an interpretation of Aristotle's explanation of how "time" is the measure ("number") of motion, without considering the many mysteries which are said to be characteristic of a kind of "time". Aristotle did not have clocks of the sort illustrated above, but one may visualize instead a sundial, an ancestor of such clocks. A sundial may be thought of as a kind of clock without a mechanism to turn the hands, making use instead of shadows.

Given a circle with length of circumference C and length of any one of its radii r (same units of length as C), a radius may be rotated about the center of the circle, say counterclockwise, so



the total

that the new position and old position of the radius form an angle. The angle formed by two such positions of a radius will determine an arc of length s on the circumference. The angle is then said to be s/r radians in size, or more simply, since s/r is just a number without physical dimension, the measure of the angle is said to be s/r radians. The ratio C/r of length of the circumference C of a circle to the length of its radius r is 2π . Thus the angle formed by one complete revolution of a radius is 2π radians = 360° . Given two circles with radii r_1 and r_2 which when rotated cut off arcs s_1 and s_2 in such a way that $s_1/r_1 = s_2/r_2$, the angles formed in the two circles will be equal, i.e. in the above figure $\theta_1 = \theta_2$.

Thus if the two circular disks in the figure represent a clock and an astronomical observation of the sun's motion from the earth, one may suppose that the mechanism of the clock is made in such a way that the clock's hands trace out the same size angles as the sun's observed positions do. More simply, if the smaller circle represents a sundial, then the tips of the shadows cast by the gnomon will necessarily trace out the same size angles. (The *gnomon* is the upright part of the sundial; see the illustration of a sundial above.)

Consider such a circular clock with length of circumference C and length of radius r . Successively rotate the longer hand of this clock 60 times through an angle of $2\pi/60$ radians = 6° . Let s denote the length of arc cut off on the circumference by a radius formed by the long hand during one of these rotations. Then for each such rotation there is an observable change of position of the hand, measured by the angle formed by the old and new positions of the hand. The size of this angle is s/r radians.

I will now introduce some terms which are commonly used to designate units of "time", but defined in terms of angular measure rather than in some other way. Set $1 H = 2\pi = 360^\circ$, so that the angle for one complete rotation measures 1 H. Also define a unit M by $1 M = H/60 = 2\pi/60 = 6^\circ$, so that $60 M = 1 H$.

Suppose, for example, a vehicle travels 90 km, and during the trip the hand of a particular clock rotates through an angle measuring 3π radians = 540° . Set $H = 2\pi$ radians = 360° , which measures the angle swept out during one revolution of a long hand or a gnomon's shadow on our clock. Then the average *rate* at which the vehicle traveled may be said to be $90 \text{ km}/1.5H = 60 \text{ km}/H = 60 \text{ km}/2\pi \text{ radians} = 30 \text{ km}/\pi \text{ radians}$. This may be read as 30 km per each angular change of size π radians = 180° , or if preferred, as 60 km per each complete rotation of the longer hand of the clock being used.

Suppose we call each revolution H , measured in radians, an *hour*, and each $H/60 = M$ a *minute*. That is, suppose we call 2π radians or 360° an *hour* and $2\pi/60$ radians or 6° a *minute*, so that there are 60 minutes in an hour. In the example above, we can then say that the average rate of change, or *speed*, of the vehicle is 60 km/hour.

Suppose further that we understand these hours and minutes *only* as referring to angular measures as determined in the way described above, and *not* as referring to something called "time", or "passage of time", or "consciousness of time", etc. Then we have defined rates of change of distances, or speeds, *not* as so many units of distance per units of some entity called "time" which flows or passes or which people pass in or experience, but as so many units of distance per revolutions of a line segment, such as the long hand of a kind of clock.

Suppose we go further and *define* a term 'time' to be used in the following way. That is, suppose we say that this kind of time is a measure of fractions of revolutions of the radii of circles. Of course, there would still be problems with timekeeping arising from the fact that the orbits of celestial objects are not *exactly* circles.

5. Conclusion.

We then have, as Aristotle proposed, time as a measure or number of motion, where motion is measured with units of distance. Conversely, if we have 'time' *defined* this way, and speeds defined by units of distance per units of this kind of time then we can calculate the corresponding changes in distance. For example, if we know that a vehicle travels at an average speed of 60 km/hour (i.e. 60 km per 2π radians) for 1.5 hours (i.e. for 3π radians), then it will traverse a distance of 90 km. In this way, we can save the hallowed formula distance = speed x time.

So "time," defined this way, is a number of motion, and motion is a number of "time," as Aristotle said.

One can eliminate certain concerns about what sort of *thing* it is that mathematicians, physicists and others often refer to when they use the term 'time' in their work concerning motions of objects. I am claiming that here 'time' refers to *empirical* measurements of fractions of revolutions of radii of circles, which are commonly dealt with using certain mathematical *concepts* for purposes of approximation. Here we have a usage of the term 'time' which could be eliminated entirely from our language when

dealing with such matters as classical and relativistic mechanics, e.g. the kinds of time Newton and Einstein talked about. But I have also shown how I think we can still use the term 'time' in ways that Newton and Einstein and some of their predecessors and successors have used them by defining it the way I have done. We might even come to accept "time flows" as a supportable metaphor if we take it that hands of a clock and certain celestial objects flow when move.

If I am right, philosophers could dispense with questions about what a certain kind of 'time' *is*. However, there are other usages of the term 'time' they could still be concerned with, e.g. the phenomenology of internal time consciousness, as Husserl called it. There are times and then there are other times.